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A Monte Carlo Comparison of the Statistical Performance of Selected Robust Linear Regression Estimators under Varying Sample Sizes and Outlier Contamination Proportions

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مقارنة باستخدام محاكاة مونت كارلو للأداء الإحصائي لمقدِّرات الانحدار الخطي المتينة المختارة في ظل أحجام عينات ونِسَب مختلفة من تلوث البيانات بالقيم الشاذة

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Monte Carlo simulation; robust regression; Ordinary Least Squares (OLS); MM-estimation; outlier contamination.

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ABSTRACT

This study compares the statistical performance of Ordinary Least Squares (OLS), Huber M-estimation, Least Trimmed Squares (LTS), and MM-estimation under varying sample sizes and outlier contamination proportions using Monte Carlo simulation. The data-generating model contains two independent standard-normal explanatory variables, with true parameters $\beta_0 = 2$, $\beta_1 = 3$, and $\beta_2 = 2$. Four sample sizes (n = 40, 60, 100, and 200) and four contamination proportions (π = 0%, 5%, 10%, and 20%) were considered, producing sixteen simulation conditions. Vertical outliers were introduced through the regression error term, and each condition was replicated 1,000 times. Estimator performance was evaluated using bias, variance, mean squared error (MSE), root mean squared error (RMSE), and MSE-based relative efficiency. OLS produced the lowest MSE and RMSE under uncontaminated conditions, but its performance deteriorated markedly as contamination increased. The robust estimators remained more stable under contamination; Huber M-estimation was competitive at lower contamination levels, LTS became increasingly competitive as contamination increased, and MM-estimation generally produced the lowest MSE and RMSE and the highest relative efficiency under contaminated conditions. Increasing sample size improved estimation precision for all estimators but did not remove the sensitivity of OLS to outliers. These findings indicate that estimator selection should be guided by sample characteristics and the expected presence and degree of contamination.

الملخص

تهدف هذه الدراسة إلى مقارنة الأداء الإحصائي لكل من طريقة المربعات الصغرى العادية(*OLS*) ، وتقدير *HUBER* من نوع *M* ، وطريقة المربعات الأقل تقليمًا(*LTS*) ، وتقدير *MM* ، وذلك في ظل اختلاف أحجام العينات ونِسَب تلوث البيانات بالقيم الشاذة، باستخدام محاكاة مونت كارلو. يتضمن نموذج توليد البيانات متغيرين تفسيريين مستقلين يتبعان التوزيع الطبيعي المعياري، مع استخدام المعلمات الحقيقية $B_0 = 2$ و $B_1 = 3$ و $B_2 = 2.$وقد جرى النظر في أربعة أحجام للعينات($N = 40$) ، 60، 100، 200 (وأربع نسب للتلوث$\Pi = 0\%$) ، 5%، 10%، 20%(، مما نتج عنه ست عشرة حالة محاكاة. وأُدخلت القيم الشاذة الرأسية من خلال حد الخطأ في نموذج الانحدار، وتم تكرار كل حالة محاكاة 1000 مرة. وتم تقييم أداء المقدِّرات باستخدام التحيز، والتباين، ومتوسط مربع الخطأ(*MSE*) ، والجذر التربيعي لمتوسط مربع الخطأ(*RMSE*) ، والكفاءة النسبية القائمة على*.MSE* أظهرت النتائج أن طريقة المربعات الصغرى العادية (*OLS*) حققت أدنى قيم لمتوسط مربع الخطأ (*MSE*) والجذر التربيعي لمتوسط مربع الخطأ (*RMSE*) في حالات عدم وجود تلوث، إلا أن أداءها تدهور بصورة ملحوظة مع زيادة نسبة التلوث. وفي المقابل، أظهرت المقدِّرات المتينة استقرارًا أكبر في ظل وجود القيم الشاذة؛ حيث كان تقدير *HUBER* من نوع *M* منافسًا عند مستويات التلوث المنخفضة، بينما أصبحت طريقة المربعات الأقل تقليمًا (*LTS*) أكثر تنافسية مع زيادة مستوى التلوث. كما أظهر تقدير *MM* بصورة عامة أدنى قيم لمتوسط مربع الخطأ (*MSE*) والجذر التربيعي لمتوسط مربع الخطأ(*RMSE*) ، وأعلى كفاءة نسبية في ظل حالات التلوث. وأسهمت زيادة حجم العينة في تحسين دقة التقدير لجميع المقدِّرات، لكنها لم تُلغِ حساسية طريقة المربعات الصغرى العادية للقيم الشاذة. وتشير هذه النتائج إلى أن اختيار المقدِّر المناسب ينبغي أن يستند إلى خصائص العينة، مع مراعاة التوقع بوجود القيم الشاذة ودرجة انتشارها.

***الكلمات المفتاحية* :** *محاكاة مونت كارلو ، الانحدار المتين ، المربعات الصغرى* العادية(*OLS*) ، تقدير *Huber* من نوع *M* ، المربعات الأقل تقليمًا (*LTS*) ، تقدير*MM* ، تلوث البيانات بالقيم الشاذة.

1. INTRODUCTION

Linear regression is one of the most widely used statistical methods for studying the relationship between a dependent variable and one or more explanatory variables. Ordinary Least Squares (OLS) is the conventional method for estimating the parameters of a linear regression model because of its simplicity, computational convenience, and high statistical efficiency when the underlying model assumptions are satisfied. However, the least-squares criterion is based on squared residuals, which makes OLS particularly sensitive to observations with unusually large residuals. Consequently, even a relatively small number of outlying observations can exert disproportionate influence on estimated regression coefficients and substantially reduce estimation accuracy (Huber & Ronchetti, 2009; Yu & Yao, 2017). The presence of outliers is therefore an important concern in regression analysis, particularly when the objective is to obtain reliable parameter estimates from data that may contain atypical observations. The effect of outliers depends not only on their magnitude and location but also on their proportion within the sample and on the sample size. Under contaminated conditions, the performance of OLS may deteriorate considerably, whereas estimators specifically designed to reduce the influence of atypical observations may provide more stable estimates. These considerations have led to the development of robust regression methods that seek to achieve an appropriate balance between statistical efficiency under clean data and resistance to contamination. Several robust regression procedures have been developed to address the limitations of classical least-squares estimation. Huber M-estimation reduces the influence of observations with large residuals by replacing the quadratic loss function with a robust loss function that limits the contribution of extreme residuals (Huber, 1973; Huber & Ronchetti, 2009). The Least Trimmed Squares (LTS) estimator provides greater resistance to contamination by basing estimation on a subset of observations having the smallest squared residuals (Rousseeuw, 1984; Rousseeuw & Leroy, 1987). MM-estimation was subsequently developed to combine a high level of robustness with high statistical efficiency under approximately normal error distributions (Yohai, 1987). Although these methods share the general objective of reducing the influence of outlying observations, they differ in their statistical properties and may therefore perform differently under different data conditions. The comparative performance of regression estimators cannot be determined solely from their theoretical properties. It may change according to sample size, contamination proportion, contamination structure, and the criterion used to evaluate estimator performance. Simulation studies provide an appropriate framework for investigating these differences because they allow data to be generated under controlled conditions with known population parameters and allow the same estimation procedures to be repeatedly evaluated across systematically varied scenarios (Morris et al., 2019). Previous research has demonstrated that robust estimators can provide substantial advantages over OLS when data contain contamination, while OLS may retain greater efficiency when the data are uncontaminated (Huber & Ronchetti, 2009; Yu & Yao, 2017). Despite the extensive development of robust regression methods, their relative performance is not necessarily uniform across sample sizes and contamination

conditions.

Conclusions obtained from simulation studies depend on the characteristics of the simulation design, including the sample sizes considered, the proportion and type of contamination, the estimators included in the comparison, and the performance measures used to evaluate them (Morris et al., 2019). Thus, a controlled comparison in which different estimators are applied to the same generated datasets under systematically varied conditions can provide useful evidence regarding their relative performance.

Accordingly, the present study employs a Monte Carlo simulation to compare the statistical performance of OLS, Huber M-estimation, LTS, and MM-estimation under varying sample sizes and vertical outlier contamination proportions. Four sample sizes (n = 40, 60, 100, and 200) and four contamination proportions (π = 0%, 5%, 10%, and 20%) are considered, resulting in sixteen simulation conditions. Each condition is replicated 1,000 times, and estimator performance is evaluated using bias, variance, mean squared error (MSE), root mean squared error (RMSE), and MSE-based relative efficiency. The study therefore aims to provide a controlled assessment of the extent to which sample size and outlier contamination affect the relative performance of the four regression estimators and to identify the circumstances under which robust estimation provides an advantage over the classical OLS approach

2. METHOD

2.1 Research Design

This study uses a quantitative experimental design based on Monte Carlo simulation. Four regression estimators—OLS, Huber M-estimation, LTS, and MM-estimation were compared under controlled data-generating conditions. Four sample sizes and four vertical outlier contamination proportions were combined to produce sixteen simulation conditions. Within each condition, all four estimators were applied to the same generated datasets, and each condition was repeated 1,000 times. Performance was evaluated using bias, variance, MSE, RMSE, and MSE-based relative efficiency.

2.2 Data-Generating Model

The simulation is based on the following multiple linear regression model with two explanatory variables:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i \quad (1)$$

where y_i is the response, x_{i1} and x_{i2} are explanatory variables, β_0 is the intercept, β_1 and β_2 are regression coefficients, and ε_i is the random error. The true parameter values were fixed at $\beta_0 = 2$, $\beta_1 = 3$, and $\beta_2 = 2$.

$$x_{i1} \sim N(0,1),\ x_{i2} \sim N(0,1) \quad (2)$$

The explanatory variables were generated independently from standard normal distributions. Under the uncontaminated condition, the regression errors were generated as:

$$\varepsilon_i \sim N(0,1) \quad (3)$$

For contaminated observations, the error distribution was represented as $N(0, \sigma_c^2)$, where $\sigma_c^2 = 100$ denotes the fixed contamination variance used in the simulation. The contamination mixture can therefore be written as:

$$\varepsilon_i \sim (1 - \pi)N(0,1) + \pi N(0, \sigma_c^2) \quad (4)$$

2.3 Sample Sizes and Outlier Contamination Proportions

Four sample sizes were considered: n = 40, 60, 100, and 200. Four contamination proportions were considered: π = 0, 0.05, 0.10, and 0.20, corresponding to 0%, 5%, 10%, and 20%, respectively. The combinations produced sixteen simulation conditions. The numbers of contaminated observations are shown in Table 1.

Table 1. Number of Contaminated Observations by Sample Size and Contamination Proportion

Sample Size (n)	0%	5%	10%	20%
40	0	2	4	8
60	0	3	6	12
100	0	5	10	20
200	0	10	20	40

2.4 Contamination Mechanism

Vertical outliers were introduced through the regression error term, while the explanatory variables remained generated from their standard normal distributions. Within each replication, the required number of observations was randomly

selected for contamination, while the remaining observations retained the clean error distribution. The contamination proportion varied across conditions, whereas the contamination severity was held constant.

2.5 Simulation Procedure and Number of Replications

For each of the sixteen conditions, a dataset was generated according to the specified regression model and contamination mechanism. OLS, Huber M-estimation, LTS, and MM-estimation were then fitted to the same dataset, and the estimated coefficients β_0, β_1, and β_2 were recorded.

Using the same dataset for all four estimators within each replication ensured a direct comparison. Each condition was repeated R = 1,000 times, resulting in 16,000 simulation replications.

2.6 Performance Measures

Estimator performance was evaluated using the following measures.

Bias measures the average deviation of an estimated coefficient from its true population value:

$$Bias(\hat{\beta}_j) = \frac{1}{R}\sum_{r=1}^{R}(\hat{\beta}_{jr} - \beta_j) \quad (5)$$

Variance measures the dispersion of the coefficient estimates around their empirical mean:

$$Var(\hat{\beta}_j) = \frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}_{jr} - \bar{\beta}_j)^2 \quad (6)$$

Mean squared error combines squared bias and variance:

$$MSE(\hat{\beta}_j) = \frac{1}{R}\sum_{r=1}^{R}(\hat{\beta}_{jr} - \beta_j)^2 \quad (7)$$

Root mean squared error is the square root of MSE:

$$RMSE(\hat{\beta}_j) = \sqrt{MSE(\hat{\beta}_j)} \quad (8)$$

Relative efficiency compares each robust estimator with OLS using MSE as the criterion:

$$R\,E = \frac{MSE_{\boldsymbol{OLS}}(\widehat{\beta} j)}{MSE_{\boldsymbol{Robust}}(\widehat{\beta} j)} \quad (9)$$

Values of RE > 1 indicate a lower MSE for the robust estimator than for OLS; RE = 1 indicates equal MSE; and RE < 1 indicates a lower MSE for OLS.

2.7 Regression Estimation Methods

Ordinary Least Squares (OLS)

OLS estimates the regression coefficients by minimizing the sum of squared residuals:

$$\hat{\beta_{OLS}} = arg\ min\ \beta \sum_{i=1}^{n}(y_i - x_i^T\beta)^2 \quad (10)$$

OLS served as the benchmark estimator. Its squared-residual criterion makes it sensitive to observations with unusually large residuals (Huber & Ronchetti, 2009; Yu & Yao, 2017).

Huber M-Estimation

Huber M-estimation reduces the influence of large residuals by replacing the squared-error loss with a piecewise loss function (Huber, 1973; Huber & Ronchetti, 2009):

$$\hat{\beta_H} = arg\ min\ \beta \sum_{i=1}^{n}\rho H(r_i) \quad (11)$$

where r_i is the residual and $\rho c(\cdot)$ is the Huber loss function:

$$\rho_c(r) = \{\, 0.5r^2, if\ |r| <= c;\ \ c|r| - 0.5c^2, if\ |r| > c\,\} \quad (12)$$

The simulation used the Huber influence function with tuning constant k = 1.345 and MAD scale estimation.

Least Trimmed Squares (LTS)
LTS reduces the influence of outliers by minimizing a subset of the smallest squared residuals (Rousseeuw, 1984; Rousseeuw & Leroy, 1987):

$$\hat{\beta}_{LTS} = arg\ min\ \beta \sum_{i=1}^{h} r_(i:n)^2 \quad (13)$$

where $r^2(1:n) \leq . \leq r^2(n:n)$ are the ordered squared residuals and h is the number of observations retained in the objective function. The simulation used $\alpha = 0.50$ and nsamp = 500.

MM-Estimation
MM-estimation is designed to combine high breakdown robustness with relatively high efficiency under approximately normal errors (Yohai, 1987; Maronna et al., 2019). A general objective form is:

$$\hat{\beta}_{MM} = arg\ min\ \beta \sum_{i=1}^{n} \rho(r_i/s) \quad (14)$$

where s is a robust scale estimate and $\rho(\cdot)$ is a robust loss function. The simulation implemented MM-estimation using rlm() with method = "MM".

Comparison of the Estimation Methods
The four estimators represent different trade-offs between efficiency under clean data and resistance to contamination. OLS provides the benchmark under uncontaminated conditions; Huber M-estimation downweights large residuals; LTS trims observations with the largest residual contributions; and MM-estimation combines a robust initial estimate with a subsequent efficient M-estimation step. Their performance was compared to using common simulation datasets and common performance measures.

Statistical Software and Simulation Implementation
The Monte Carlo simulation was implemented in IBM SPSS Statistics using the Integration Plug-in for R and embedded R code. OLS was estimated using lm() from the R stats package. Huber M-estimation was implemented using rlm() from MASS with the Huber influence function, k = 1.345, MAD scale estimation, and a maximum of 100 iterations. LTS was estimated using ltsReg() from robustbase with $\alpha = 0.50$ and nsamp = 500. MM-estimation was implemented using rlm() from MASS with method = "MM" and a maximum of 100 iterations. Separate deterministic seeds were used for data generation and estimator-specific procedures.
Data-generation seed: 20260814; LTS seed: 120260814; MM seed: 220260814

3. ETHICS APPROVAL

Ethical approval was not applicable because the study was based entirely on Monte Carlo simulated data and did not involve human participants.

4. RESULTS

The results are presented for the sixteen combinations of sample size and contamination proportion. Estimator performance is summarized using the mean coefficient estimate, bias, variance, MSE, RMSE, and MSE-based relative efficiency. Because the simulation is descriptive and no formal inferential tests of the interaction between sample size and contamination proportion were conducted, the results are interpreted as observed simulation patterns.

Table 2. Monte Carlo Simulation Results for n = 40, π = 0

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000	2.004765	0.004765	0.027782	0.027777	0.166663	1.000000
OLS	β_1	3.000	2.995462	-0.004538	0.029703	0.029694	0.172318	1.000000
OLS	β_2	2.000	1.999115	-0.000885	0.028716	0.028688	0.169376	1.000000
Huber	β_0	2.000	2.002329	0.002329	0.028425	0.028402	0.168528	0.977997
Huber	β_1	3.000	2.995400	-0.004600	0.031424	0.031414	0.177239	0.945238
Huber	β_2	2.000	2.000854	0.000854	0.031236	0.031205	0.176651	0.919336
LTS	β_0	2.000	1.995087	-0.004913	0.041672	0.041655	0.204095	0.666830
LTS	β_1	3.000	2.996337	-0.003663	0.054366	0.054325	0.233077	0.546593
LTS	β_2	2.000	2.002929	0.002929	0.051639	0.051596	0.227148	0.556017
MM	β_0	2.000	2.002168	0.002168	0.028759	0.028735	0.169513	0.966664
MM	β_1	3.000	2.995198	-0.004802	0.031680	0.031671	0.177964	0.937556
MM	β_2	2.000	2.000461	0.000461	0.031810	0.031779	0.178266	0.902750

Under the uncontaminated conditions, OLS achieved the lowest MSE and RMSE across the three coefficients, while Huber and MM remained relatively close, and LTS showed lower clean-data efficiency.

Table 3. Monte Carlo Simulation Results for n = 40, π = 0.05

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.973667	-0.026333	0.150968	0.151511	0.389244	1.000000
OLS	β_1	3.000000	2.994944	-0.005056	0.150973	0.150848	0.388391	1.000000
OLS	β_2	2.000000	2.042915	0.042915	0.153417	0.155105	0.393833	1.000000
Huber	β_0	2.000000	1.992595	-0.007405	0.030366	0.030390	0.174328	4.985482
Huber	β_1	3.000000	3.002734	0.002734	0.036389	0.036361	0.190684	4.148669
Huber	β_2	2.000000	2.012667	0.012667	0.033390	0.033517	0.183076	4.627652
LTS	β_0	2.000000	1.999800	-0.000200	0.042602	0.042560	0.206300	3.559941
LTS	β_1	3.000000	3.000632	0.000632	0.050204	0.050154	0.223951	3.007687
LTS	β_2	2.000000	2.007611	0.007611	0.052265	0.052271	0.228628	2.967330
MM	β_0	2.000000	1.997363	-0.002637	0.027744	0.027723	0.166503	5.465120
MM	β_1	3.000000	3.002181	0.002181	0.033343	0.033314	0.182522	4.528020
MM	β_2	2.000000	2.009046	0.009046	0.031779	0.031829	0.178408	4.873028

At 5% contamination, the robust estimators substantially outperformed OLS. MM-estimation produced the lowest MSE across the three coefficients in this condition.

Table 4. Monte Carlo Simulation Results for n = 40, π = 0.10

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.980497	-0.019503	0.295569	0.295654	0.543741	1.000000
OLS	β_1	3.000000	3.013770	0.013770	0.273041	0.272958	0.522454	1.000000
OLS	β_2	2.000000	2.015044	0.015044	0.283391	0.283334	0.532291	1.000000
Huber	β_0	2.000000	1.996829	-0.003171	0.040031	0.040001	0.200002	7.391221
Huber	β_1	3.000000	3.001286	0.001286	0.043396	0.043355	0.208218	6.295927
Huber	β_2	2.000000	2.013830	0.013830	0.040848	0.040999	0.202481	6.910805
LTS	β_0	2.000000	2.000528	0.000528	0.042582	0.042540	0.206252	6.950043
LTS	β_1	3.000000	2.999906	-0.000094	0.048746	0.048697	0.220674	5.605221
LTS	β_2	2.000000	2.013450	0.013450	0.045941	0.046076	0.214653	6.149290
MM	β_0	2.000000	2.000266	0.000266	0.035077	0.035042	0.187194	8.437214
MM	β_1	3.000000	3.000926	0.000926	0.035818	0.035783	0.189164	7.628137
MM	β_2	2.000000	2.012561	0.012561	0.033015	0.033140	0.182044	8.549585

At 10% contamination, the performance gap between OLS and the robust estimators widened. MM-estimation produced the lowest MSE across the three coefficients.

Table 5. Monte Carlo Simulation Results for n = 40, $\pi = 0.20$

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.993699	-0.006301	0.532282	0.531789	0.729239	1.000000
OLS	β_1	3.000000	3.036653	0.036653	0.581424	0.582186	0.763011	1.000000
OLS	β_2	2.000000	2.026449	0.026449	0.548559	0.548710	0.740750	1.000000
Huber	β_0	2.000000	1.998980	-0.001020	0.058805	0.058748	0.242379	9.052104
Huber	β_1	3.000000	3.017077	0.017077	0.069826	0.070048	0.264665	8.311275
Huber	β_2	2.000000	2.009120	0.009120	0.070745	0.070757	0.266002	7.754844
LTS	β_0	2.000000	2.001774	0.001774	0.043455	0.043415	0.208363	12.248930
LTS	β_1	3.000000	3.014821	0.014821	0.053417	0.053583	0.231480	10.865085
LTS	β_2	2.000000	1.995689	-0.004311	0.055659	0.055622	0.235843	9.864977
MM	β_0	2.000000	2.001195	0.001195	0.041472	0.041432	0.203548	12.835289
MM	β_1	3.000000	3.013883	0.013883	0.046755	0.046901	0.216566	12.413121
MM	β_2	2.000000	2.002137	0.002137	0.050121	0.050075	0.223775	10.957730

At 20% contamination, MM-estimation again produced the lowest MSE across the three coefficients, while LTS was highly competitive and outperformed Huber.

Table 6. Monte Carlo Simulation Results for n = 60, $\pi = 0$

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	2.003513	0.003513	0.017705	0.017699	0.133039	1.000000
OLS	β_1	3.000000	3.002652	0.002652	0.017636	0.017625	0.132760	1.000000
OLS	β_2	2.000000	2.002236	0.002236	0.017430	0.017418	0.131977	1.000000
Huber	β_0	2.000000	2.002706	0.002706	0.018707	0.018695	0.136730	0.946739
Huber	β_1	3.000000	3.003605	0.003605	0.018943	0.018937	0.137610	0.930752
Huber	β_2	2.000000	2.001772	0.001772	0.018574	0.018558	0.136229	0.938551
LTS	β_0	2.000000	2.000087	0.000087	0.026948	0.026921	0.164078	0.657446
LTS	β_1	3.000000	3.004469	0.004469	0.031684	0.031672	0.177966	0.556499
LTS	β_2	2.000000	2.001991	0.001991	0.029617	0.029592	0.172022	0.588609
MM	β_0	2.000000	2.002721	0.002721	0.018680	0.018668	0.136632	0.948094
MM	β_1	3.000000	3.003520	0.003520	0.018987	0.018980	0.137768	0.928621
MM	β_2	2.000000	2.002606	0.002606	0.018669	0.018657	0.136592	0.933567

At n = 60 and $\pi = 0$, OLS retained the lowest MSE and RMSE, while Huber and MM remained close and LTS showed lower clean-data efficiency.

Table 7. Monte Carlo Simulation Results for n = 60, $\pi = 0.05$

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.985717	-0.014283	0.098674	0.098779	0.314291	1.000000
OLS	β_1	3.000000	3.000962	0.000962	0.109189	0.109081	0.330273	1.000000
OLS	β_2	2.000000	2.007364	0.007364	0.101940	0.101892	0.319206	1.000000
Huber	β_0	2.000000	1.990337	-0.009663	0.021607	0.021679	0.147237	4.556485
Huber	β_1	3.000000	2.999927	-0.000073	0.021616	0.021594	0.146950	5.051338
Huber	β_2	2.000000	1.999242	-0.000758	0.023500	0.023477	0.153222	4.340076
LTS	β_0	2.000000	1.993112	-0.006888	0.026459	0.026480	0.162725	3.730389
LTS	β_1	3.000000	3.001820	0.001820	0.027879	0.027855	0.166897	3.916050
LTS	β_2	2.000000	1.998286	-0.001714	0.029711	0.029685	0.172292	3.432494
MM	β_0	2.000000	1.991167	-0.008833	0.020274	0.020332	0.142591	4.858240
MM	β_1	3.000000	3.000395	0.000395	0.019532	0.019512	0.139687	5.590326
MM	β_2	2.000000	1.998948	-0.001052	0.021253	0.021233	0.145714	4.798871

At n = 60 and $\pi = 0.05$, the robust estimators substantially outperformed OLS, with MM-estimation producing the lowest MSE across the three coefficients.

Table 8. Monte Carlo Simulation Results for n = 60, π = 0.10

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.991996	-0.008004	0.188081	0.187957	0.433540	1.000000
OLS	β_1	3.000000	2.986288	-0.013712	0.187345	0.187346	0.432835	1.000000
OLS	β_2	2.000000	1.991432	-0.008568	0.185507	0.185395	0.430575	1.000000
Huber	β_0	2.000000	2.004975	0.004975	0.026310	0.026308	0.162198	7.144397
Huber	β_1	3.000000	2.999467	-0.000533	0.026317	0.026291	0.162144	7.125953
Huber	β_2	2.000000	1.998193	-0.001807	0.025529	0.025506	0.159707	7.268603
LTS	β_0	2.000000	2.003281	0.003281	0.025789	0.025774	0.160542	7.292606
LTS	β_1	3.000000	3.001433	0.001433	0.028516	0.028489	0.168788	6.575981
LTS	β_2	2.000000	1.996729	-0.003271	0.028890	0.028872	0.169918	6.421236
MM	β_0	2.000000	2.004891	0.004891	0.021662	0.021664	0.147188	8.675840
MM	β_1	3.000000	3.002127	0.002127	0.022088	0.022071	0.148563	8.488376
MM	β_2	2.000000	1.997451	-0.002549	0.021250	0.021235	0.145723	8.730529

At n = 60 and π = 0.10, MM-estimation produced the lowest MSE across the three coefficients, while Huber and LTS remained substantially more stable than OLS.

Table 9. Monte Carlo Simulation Results for n = 60, π = 0.20

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.987888	-0.012112	0.380831	0.380597	0.616925	1.000000
OLS	β_1	3.000000	2.984876	-0.015124	0.373744	0.373599	0.611227	1.000000
OLS	β_2	2.000000	1.985721	-0.014279	0.401629	0.401431	0.633586	1.000000
Huber	β_0	2.000000	2.001435	0.001435	0.040279	0.040241	0.200602	9.457923
Huber	β_1	3.000000	2.998145	-0.001855	0.041862	0.041824	0.204508	8.932715
Huber	β_2	2.000000	2.002607	0.002607	0.042389	0.042353	0.205798	9.478230
LTS	β_0	2.000000	2.004882	0.004882	0.029553	0.029548	0.171894	12.880807
LTS	β_1	3.000000	3.002554	0.002554	0.032749	0.032723	0.180895	11.417070
LTS	β_2	2.000000	2.006454	0.006454	0.032676	0.032684	0.180789	12.282015
MM	β_0	2.000000	2.002968	0.002968	0.028808	0.028788	0.169672	13.220452
MM	β_1	3.000000	3.003312	0.003312	0.029097	0.029079	0.170526	12.847633
MM	β_2	2.000000	2.003787	0.003787	0.029566	0.029551	0.171904	13.584405

At n = 60 and π = 0.20, MM-estimation again produced the lowest MSE overall, while LTS became particularly competitive.

Table 10. Monte Carlo Simulation Results for n = 100, π = 0

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	2.000652	0.000652	0.010206	0.010196	0.100974	1.000000
OLS	β_1	3.000000	3.004786	0.004786	0.011302	0.011314	0.106367	1.000000
OLS	β_2	2.000000	1.993879	-0.006121	0.010115	0.010143	0.100711	1.000000
Huber	β_0	2.000000	2.001127	0.001127	0.010908	0.010898	0.104395	0.935539
Huber	β_1	3.000000	3.005117	0.005117	0.012043	0.012057	0.109806	0.938347
Huber	β_2	2.000000	1.994021	-0.005979	0.010776	0.010801	0.103926	0.939088
LTS	β_0	2.000000	2.000401	0.000401	0.015635	0.015619	0.124977	0.652762
LTS	β_1	3.000000	3.004936	0.004936	0.018488	0.018494	0.135992	0.611776
LTS	β_2	2.000000	1.992724	-0.007276	0.016477	0.016514	0.128506	0.614197
MM	β_0	2.000000	2.001072	0.001072	0.010913	0.010903	0.104416	0.935152
MM	β_1	3.000000	3.005163	0.005163	0.012114	0.012128	0.110128	0.932878
MM	β_2	2.000000	1.994153	-0.005847	0.010876	0.010899	0.104399	0.930600

At n = 100 and π = 0, OLS achieved the lowest MSE and RMSE, whereas Huber and MM remained close, and LTS showed lower efficiency.

Table 11. Monte Carlo Simulation Results for n = 100, $\pi = 0.05$

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.985182	-0.014818	0.057374	0.057536	0.239867	1.000000
OLS	β_1	3.000000	2.991063	-0.008937	0.064651	0.064666	0.254295	1.000000
OLS	β_2	2.000000	1.999751	-0.000249	0.061799	0.061737	0.248470	1.000000
Huber	β_0	2.000000	1.995906	-0.004094	0.011867	0.011872	0.108956	4.846562
Huber	β_1	3.000000	3.000499	0.000499	0.014413	0.014399	0.119995	4.491043
Huber	β_2	2.000000	1.996823	-0.003177	0.013726	0.013722	0.117143	4.498999
LTS	β_0	2.000000	1.997883	-0.002117	0.014123	0.014114	0.118802	4.076577
LTS	β_1	3.000000	3.003893	0.003893	0.017487	0.017485	0.132231	3.698356
LTS	β_2	2.000000	1.994324	-0.005676	0.016949	0.016965	0.130248	3.639177
MM	β_0	2.000000	1.997969	-0.002031	0.010988	0.010981	0.104790	5.239671
MM	β_1	3.000000	3.002368	0.002368	0.013052	0.013045	0.114214	4.957220
MM	β_2	2.000000	1.995600	-0.004400	0.012379	0.012386	0.111293	4.984338

At n = 100 and $\pi = 0.05$, the robust estimators substantially outperformed OLS, with MM-estimation producing the lowest MSE across the three coefficients.

Table 12. Monte Carlo Simulation Results for n = 100, $\pi = 0.10$

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.987328	-0.012672	0.115944	0.115988	0.340570	1.000000
OLS	β_1	3.000000	2.999628	-0.000372	0.126691	0.126565	0.355759	1.000000
OLS	β_2	2.000000	1.997742	-0.002258	0.123792	0.123673	0.351672	1.000000
Huber	β_0	2.000000	2.002800	0.002800	0.015487	0.015479	0.124414	7.493279
Huber	β_1	3.000000	3.001386	0.001386	0.015263	0.015250	0.123490	8.299480
Huber	β_2	2.000000	2.000718	0.000718	0.016168	0.016152	0.127092	7.656689
LTS	β_0	2.000000	2.004459	0.004459	0.015392	0.015397	0.124083	7.533336
LTS	β_1	3.000000	3.000768	0.000768	0.015943	0.015928	0.126204	7.946268
LTS	β_2	2.000000	1.997108	-0.002892	0.015913	0.015906	0.126118	7.775401
MM	β_0	2.000000	2.004806	0.004806	0.013320	0.013330	0.115454	8.701536
MM	β_1	3.000000	3.002304	0.002304	0.013365	0.013357	0.115572	9.475652
MM	β_2	2.000000	1.999187	-0.000813	0.013210	0.013197	0.114878	9.371276

At n = 100 and $\pi = 0.10$, MM-estimation provided the lowest MSE, while Huber and LTS showed similar error levels and remained more stable than OLS.

Table 13. Monte Carlo Simulation Results for n = 100, $\pi = 0.20$

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	2.011188	0.011188	0.219549	0.219454	0.468460	1.000000
OLS	β_1	3.000000	3.002276	0.002276	0.206476	0.206275	0.454175	1.000000
OLS	β_2	2.000000	1.987107	-0.012893	0.192551	0.192524	0.438776	1.000000
Huber	β_0	2.000000	1.999909	-0.000091	0.024317	0.024292	0.155860	9.033946
Huber	β_1	3.000000	2.996353	-0.003647	0.022424	0.022415	0.149716	9.202572
Huber	β_2	2.000000	2.001828	0.001828	0.022943	0.022923	0.151404	8.398675
LTS	β_0	2.000000	1.998212	-0.001788	0.018791	0.018775	0.137023	11.688406
LTS	β_1	3.000000	2.994892	-0.005108	0.016745	0.016754	0.129438	12.311915
LTS	β_2	2.000000	2.008268	0.008268	0.018103	0.018153	0.134733	10.605627
MM	β_0	2.000000	1.997909	-0.002091	0.018117	0.018104	0.134549	12.122178
MM	β_1	3.000000	2.995760	-0.004240	0.015935	0.015937	0.126242	12.943208
MM	β_2	2.000000	2.006664	0.006664	0.016991	0.017018	0.130453	11.312974

At n = 100 and $\pi = 0.20$, MM-estimation achieved the lowest MSE across the three coefficients, with LTS performing very closely.

Table 14. Monte Carlo Simulation Results for n = 200, π = 0

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	2.001905	0.001905	0.004838	0.004836	0.069544	1.000000
OLS	β_1	3.000000	2.997414	-0.002586	0.004650	0.004652	0.068206	1.000000
OLS	β_2	2.000000	1.999588	-0.000412	0.005052	0.005048	0.071046	1.000000
Huber	β_0	2.000000	2.001974	0.001974	0.005114	0.005113	0.071507	0.945866
Huber	β_1	3.000000	2.997267	-0.002733	0.004942	0.004945	0.070319	0.940792
Huber	β_2	2.000000	1.999972	-0.000028	0.005115	0.005110	0.071486	0.987740
LTS	β_0	2.000000	2.001306	0.001306	0.007027	0.007022	0.083796	0.688775
LTS	β_1	3.000000	2.995162	-0.004838	0.007093	0.007109	0.084315	0.654386
LTS	β_2	2.000000	2.001164	0.001164	0.006894	0.006888	0.082997	0.732760
MM	β_0	2.000000	2.001871	0.001871	0.005119	0.005117	0.071533	0.945165
MM	β_1	3.000000	2.997024	-0.002976	0.004951	0.004955	0.070392	0.938839
MM	β_2	2.000000	1.999987	-0.000013	0.005154	0.005149	0.071755	0.980356

At n = 200 and π = 0, OLS achieved the lowest MSE and RMSE across the coefficients, while Huber and MM remained highly competitive.

Table 15. Monte Carlo Simulation Results for n = 200, π = 0.05

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.999231	-0.000769	0.030763	0.030733	0.175309	1.000000
OLS	β_1	3.000000	2.998171	-0.001829	0.028957	0.028932	0.170093	1.000000
OLS	β_2	2.000000	2.007064	0.007064	0.031389	0.031407	0.177220	1.000000
Huber	β_0	2.000000	2.002192	0.002192	0.006516	0.006514	0.080711	4.717837
Huber	β_1	3.000000	2.998482	-0.001518	0.006224	0.006220	0.078866	4.651492
Huber	β_2	2.000000	1.998525	-0.001475	0.006364	0.006360	0.079750	4.938154
LTS	β_0	2.000000	2.001487	0.001487	0.007262	0.007257	0.085187	4.235092
LTS	β_1	3.000000	3.000825	0.000825	0.007580	0.007574	0.087026	3.820085
LTS	β_2	2.000000	1.997411	-0.002589	0.007531	0.007530	0.086778	4.170714
MM	β_0	2.000000	2.002851	0.002851	0.005827	0.005829	0.076347	5.272521
MM	β_1	3.000000	2.999215	-0.000785	0.005813	0.005808	0.076207	4.981680
MM	β_2	2.000000	1.997315	-0.002685	0.005791	0.005792	0.076108	5.422113

At n = 200 and π = 0.05, MM-estimation achieved the lowest MSE across the three coefficients, followed closely by Huber, while OLS showed substantially larger errors.

Table 16. Monte Carlo Simulation Results for n = 200, π = 0.10

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	1.994173	-0.005827	0.055541	0.055520	0.235626	1.000000
OLS	β_1	3.000000	2.990247	-0.009753	0.055372	0.055412	0.235397	1.000000
OLS	β_2	2.000000	2.006907	0.006907	0.059315	0.059304	0.243523	1.000000
Huber	β_0	2.000000	1.998373	-0.001627	0.007336	0.007331	0.085621	7.573342
Huber	β_1	3.000000	2.998682	-0.001318	0.007119	0.007114	0.084343	7.789392
Huber	β_2	2.000000	2.004124	0.004124	0.007640	0.007649	0.087458	7.753197
LTS	β_0	2.000000	1.998326	-0.001674	0.006945	0.006941	0.083314	7.998523
LTS	β_1	3.000000	3.000662	0.000662	0.007188	0.007181	0.084740	7.716640
LTS	β_2	2.000000	2.003093	0.003093	0.007306	0.007308	0.085488	8.114719
MM	β_0	2.000000	1.999175	-0.000825	0.006062	0.006056	0.077823	9.167008
MM	β_1	3.000000	3.000393	0.000393	0.005923	0.005917	0.076922	9.364959
MM	β_2	2.000000	2.002561	0.002561	0.006204	0.006204	0.078768	9.558208

At n = 200 and π = 0.10, MM-estimation again provided the lowest MSE across the three coefficients, while Huber and LTS remained substantially better than OLS.

Table 17. Monte Carlo Simulation Results for n = 200, $\pi = 0.20$

Estimator	Parameter	True Value	Mean Estimate	Bias	Variance	MSE	RMSE	Relative Efficiency
OLS	β_0	2.000000	2.003561	0.003561	0.102649	0.102559	0.320248	1.000000
OLS	β_1	3.000000	2.975653	-0.024347	0.114075	0.114554	0.338458	1.000000
OLS	β_2	2.000000	1.991470	-0.008530	0.103570	0.103540	0.321776	1.000000
Huber	β_0	2.000000	2.001690	0.001690	0.011123	0.011115	0.105426	9.227412
Huber	β_1	3.000000	2.992593	-0.007407	0.011565	0.011608	0.107740	9.868612
Huber	β_2	2.000000	1.997740	-0.002260	0.010145	0.010140	0.100700	10.210576
LTS	β_0	2.000000	1.999711	-0.000289	0.008057	0.008049	0.089717	12.741665
LTS	β_1	3.000000	2.995143	-0.004857	0.008729	0.008744	0.093507	13.101605
LTS	β_2	2.000000	1.999370	-0.000630	0.007926	0.007918	0.088985	13.075995
MM	β_0	2.000000	2.000330	0.000330	0.008089	0.008081	0.089893	12.691866
MM	β_1	3.000000	2.997383	-0.002617	0.008497	0.008495	0.092168	13.485015
MM	β_2	2.000000	1.998905	-0.001095	0.007443	0.007437	0.086235	13.923109

At n = 200 and $\pi = 0.20$, LTS achieved the lowest MSE for β_0, whereas MM-estimation achieved the lowest MSE for β_1 and β_2. Thus, the results show that no estimator was uniformly best for every coefficient under every condition.

Evaluation of Research Hypotheses

The hypotheses were assessed descriptively from the observed patterns in bias, variance, MSE, RMSE, and relative efficiency. Because no formal inferential tests were conducted, the evaluations indicate consistency with the simulated patterns rather than statistical significance.

H1: The observed simulation results are consistent with the hypothesis: increasing sample size generally reduced variance, MSE, and RMSE.

H2: Consistent with the observed simulation results: estimator performance changed as contamination proportion increased.

H3: The observed simulation results are consistent with the hypothesis: robust estimators generally produced lower MSE and RMSE than OLS under contamination.

H4: Generally consistent with the observed simulation results: the relative performance of Huber, LTS, and MM varied across contamination levels, with MM generally strongest overall and LTS particularly competitive at higher contamination.

Table 18. Evaluation of Research Hypotheses

Hypothesis	Descriptive Evaluation
H1	Consistent with the observed simulation results
H2	Consistent with the observed simulation results
H3	Consistent with the observed simulation results
H4	Generally consistent with the observed simulation results

5. DISCUSSION

Overall Pattern of Estimator Performance

Across the sixteen simulation conditions, estimator performance varied systematically with sample size and contamination proportion. Under uncontaminated conditions, OLS consistently produced the lowest MSE and RMSE. Once contamination was introduced, the robust estimators became substantially more stable, with MM-estimation generally producing the lowest MSE and RMSE across contaminated conditions.

Performance Under Uncontaminated Conditions

The uncontaminated results illustrate the expected efficiency robustness trade-off. OLS performed best under clean data, while Huber M-estimation and MM-estimation remained relatively close. LTS generally showed higher variance and MSE, which is consistent with the loss of clean-data efficiency associated with trimming observations from the objective function (Rousseeuw, 1984; Rousseeuw & Leroy, 1987).

Effect of Outlier Contamination
Introducing vertical contamination led to a marked deterioration in OLS performance. Variance, MSE, and RMSE increased as the contamination proportion rose, whereas the robust estimators maintained lower error levels. Because the clean and contaminated error distributions were specified as symmetric and centered at zero, contamination primarily increased estimation variability rather than producing a persistent directional bias. This explains why MSE and RMSE provide more informative overall measures than bias alone in the present design.

Effect of Sample Size
Increasing the sample size generally reduced variance, MSE, and RMSE for all estimators, indicating improved estimation precision. Nevertheless, larger samples did not eliminate the sensitivity of OLS to contamination. Thus, sample size and robustness address different aspects of estimator performance: larger samples improve precision, whereas robust procedures protect atypical observations.

Relative Efficiency
Using OLS as the reference, relative efficiency was approximately 1 under clean conditions for OLS by definition. Under contamination, the robust estimators frequently produced relative-efficiency values above 1, indicating lower MSE than OLS. MM-estimation generally showed the highest relative efficiency across contaminated conditions, although LTS was highly competitive at higher contamination levels and was best for β_0 in the $n = 200$, $\pi = 0.20$ condition.

Evaluation of the Simulation Hypotheses
The observed patterns were consistent with H1 and H2 in the descriptive simulation results. H3 was also consistent with the results because the robust estimators generally had lower MSE and RMSE than OLS under contamination. H4 was generally consistent with the results, although the coefficient-specific comparison at $n = 200$ and $\pi = 0.20$ shows that MM-estimation was not uniformly superior for every coefficient. These findings reinforce the importance of reporting multiple performance measures rather than relying on a single estimator ranking.

6. CONCLUSION

This Monte Carlo study compared OLS, Huber M-estimation, LTS, and MM-estimation across four sample sizes and four vertical outlier contamination proportions. OLS provided the lowest MSE and RMSE under uncontaminated conditions, whereas its performance deteriorated as contamination increased. The robust estimators remained more stable under contamination, with MM-estimation generally producing the lowest MSE and RMSE, while LTS became increasingly competitive at higher contamination.
Increasing sample size generally improved estimation precision by reducing variance, MSE, and RMSE, but it did not remove OLS sensitivity to outliers. The results therefore indicate an efficiency–robustness trade-off rather than a universally best estimator. Estimator choice should be based on the expected data characteristics and the possibility of contamination.

Limitations and Future Research
Several limitations should be considered. First, the study examined only vertical outliers introduced through the error term; leverage and bad-leverage contamination were not investigated. Second, contamination severity was held constant while only the contamination proportion varied. Third, the simulation used four sample sizes and a relatively simple linear model with two independent standard-normal predictors. More complex predictor structures, multicollinearity, nonlinear relationships, and model misspecification were not examined. Fourth, the error distributions were symmetric and centered at zero, so the findings may differ under skewed, heavy-tailed, heteroscedastic, or systematically shifted contamination. Finally, only four estimators were compared and no formal inferential tests of sample-size-by-contamination interactions were conducted.
Future research should therefore examine leverage and combined contamination, vary contamination magnitude as well as contamination proportion, consider larger and correlated predictor sets, and compare additional robust estimators and performance criteria. Empirical applications using real datasets with known or suspected outliers would also help assess the extent to which the simulation patterns generalize to practical regression settings.

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