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Zero Centric Arithmetic: Rethinking Division by Zero

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ABSTRACT

Abstracts are This paper redefines the concept of division by zero, a long-standing controversy in mathematics. In traditional field theory, division is defined as multiplication by the inverse, and since zero has no inverse, division by zero is treated as an exception and labeled "undefined." This approach, however, creates contradictions and confusion, especially when ad-hoc definitions such as $1 \div 0 = \infty$ are introduced. The paper proposes a new framework called Zero-Centric Arithmetic: Rethinking Division by Zero which interprets division as distribution. If the divisor is zero, there are no groups to distribute into, meaning the operation itself does not exist, rather than being undefined. This perspective treats zero not as a deficient element but as the central element of the number system, removing philosophical and educational ambiguities. Through practical examples (gift distribution, line slopes in geometry, limits in calculus), the paper demonstrates that interpreting division by zero as a non-existent operation is more consistent and intuitive.

1. INTRODUCTION

Division by zero has long been a controversial topic in mathematics. In traditional field theory, division is defined as:

$$a \div b = a \times b^{-1}, b \neq 0 \quad (1)$$

This requirement is usually presented as an exception, which creates confusion among learners, leads to inaccurate results, and forces illogical assumptions in many disciplines such as physics, engineering, and computer science. It even results at times in contradictions when ad-hoc definitions such as:

$$1 \div 0 = \infty \quad (2)$$

are introduced.

This paper proposes a reformulation based on the simple logic of distribution, where zero is treated as a central element that does not participate in division at all not as an undefined exception but as a non-existent operation. Instead of forcing a symbolic value at b = 0, the approach redirects attention to the limit process:

$$\lim_{b\to 0}\left(\frac{a}{b}\right) \qquad (3)$$

2. LITERATURE REVIEW

This section situates the Zero-Centric view amid prior work: classical field theory forbids a reciprocal for zero; extended systems introduce for limits without legitimizing a ÷ 0; and alternative calculi permit it only by redefining core laws. Pedagogically, the Libyan American Abacus frames division as distribution, clarifying why a zero divisor yields no operation.

2.1 Traditional Field Theory

In conventional algebra, division is defined as shown in Equation (1). Since zero has no multiplicative inverse, division by zero is therefore classified as undefined. As Neely (2022) emphasizes, any attempt to assign a multiplicative inverse to zero collapses the algebraic structure every number reduces to zero. Kaufman (2024) further argues that division by zero should not be called undefined, but rather inconsistent, as it produces contradictory mathematical statements.

2.2 Extended Number Systems

Various extensions have attempted to incorporate division by zero indirectly. The extended real line introduces $+\infty$ and $-\infty$ for limits, but not as valid results of division. The Riemann sphere (extended complex plane) sets $\frac{1}{0} = \infty$ for analytic convenience, yet $\frac{0}{0}$ remains undefined and arithmetic with ∞ is heavily restricted (Tafadzwa, 2022). These approaches clarify that division by zero cannot be assimilated without sacrificing consistency.

2.3 Alternative Approaches

Several unconventional systems, such as Transreal Arithmetic (Anderson, 2004) and Wheel Theory (Carlström, 2004), attempt to extend division by zero by redefining algebraic rules. Similarly, Abubakr (2011) introduces Calpanic Numbers as a logical extension of algebraic division, generating new entities that emerge from division by zero. However, these frameworks sacrifice familiar algebraic laws or require entirely new number systems, leading to non-standard behavior and limited adoption.

2.4 Educational Perspectives

Karakus, Güler, and Aydin (2019) show that most teachers recognize division by zero as undefined but struggle to provide conceptual reasoning. This reveals a gap in pedagogical clarity. A simple, logically consistent explanation is therefore necessary for effective teaching.

Recent educational innovation that supports this conceptual clarity is the Libyan American Abacus (Zargelin, 2025), which provides a tangible and visual model of arithmetic operations, including division. Through its physical bead activation mechanism, the abacus demonstrates how division functions as a process of distribution and crucially, how division by zero results in no operation because no beads can be activated to form groups. This pedagogical visualization directly aligns with the Zero-Centric interpretation by transforming an abstract limitation into an observable fact.

3. THE NEW FRAMEWORK: ZERO-CENTRIC ARITHMETIC

Zero-Centric Arithmetic provides a reinterpretation of division by zero as a non-existent operation rather than an undefined or infinite one. In this framework, zero is not seen as a problematic gap in mathematics but as the central anchor of the number system, incapable of division yet essential for structure. By defining division as distribution, the framework eliminates the logical contradictions of forcing infinity into arithmetic and avoids speculative constructs. It clarifies special cases such as $0 \div 0$ by treating them not as indeterminate but as operations that never occur, since there are neither objects to divide nor groups to receive them. This approach offers coherence, pedagogical clarity, and philosophical consistency, positioning zero as the stabilizing center of arithmetic rather than its exception.

3.1 Towards Zero-Centric Arithmetic

Against this background, the present paper introduces Zero-Centric Arithmetic, which reframes division by zero not as undefined or inconsistent, nor as an opportunity to invent new number systems, but as a non-existent operation. Zero is understood as the central element of the number system: it neither divides nor can be divided upon.

This approach:

1. Removes contradictions (Kaufman, Neely).
2. Avoids illogical assumptions across disciplines (Tafadzwa).
3. Provides pedagogical simplicity (Karakus et al.).
4. Rejects speculative number systems (Abubakr).

In particular, it avoids artificial assumptions such as Equation (2), where infinity is forced into arithmetic without logical basis. Thus, Zero-Centric Arithmetic positions itself as a coherent, educationally accessible, and philosophically sound resolution to a problem that has long challenged mathematics.

3.2 Core Hypothesis

Division as distribution. Division is interpreted as distribution: a $\div$ b means distributing a among b groups.

The case of zero groups. If $b = 0$, there are no groups at all. This interpretation connects back to Equation (3), which traditionally relies on limits rather than defining division at $b = 0$.

Conclusion. The operation is therefore non-existent (No Operation), not undefined and not infinite.

3.3 Zero as a Center

- Zero represents the absence of the divisor, not a failure of mathematics.
- There is no need to assume a reciprocal for zero, because zero simply does not participate in division.
- Zero is the central element of the number system not a deficient element lacking an inverse, but the anchor around which arithmetic is structured.

3.4 The Case of 0 $\div$ 0

In traditional mathematics, $0 \div 0$ is treated as indeterminate. Algebraically, if we assume $0 \div 0 = c$, then multiplying both sides by 0 yields$0 \div 0$, which is true for any value of c. This opens the possibility of infinitely many answers, hence indeterminacy. In calculus, the form $\left(\frac{0}{0}\right)$ often appears in limits, but different contexts lead to different results:

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1 \qquad (5)$$

$$\lim_{x\to 0}\left(\frac{x}{x}\right) = 1 \qquad (6)$$

$$\lim_{x\to 0}\left(\frac{x^2}{x}\right) = 0 \qquad (7)$$

As Equations (5) – (7) illustrate, the same symbolic form $\left(\frac{0}{0}\right)$ produces different outcomes, reinforcing the label indeterminate. .

Within Zero-Centric Arithmetic, however, $0 \div 0$ is interpreted differently. Division represents distribution:

$a \div b$ ameans distributing objects into b groups. If both $a = 0$ and $b = 0$, there are no objects to distribute and no groups to receive them. Hence, the operation itself does not exist. .

Thus, in this framework, $0 \div 0$ is not indeterminate, but non-existent. It is not a value waiting to be determined, nor an infinite outcome. It is the absence of an operation altogether, consistent with the principle that division by zero never arises.

3. ETHIC APPROVAL

This study is purely theoretical/conceptual. It involves no human participants, no animals, no personal data, and no field interventions. Therefore, ethics committee review was not required. Institutional permissions & IP: Official letters from the Libyan Authority for Scientific Research and the Ministry of Culture and Cognitive Development (attached) confirm institutional endorsement and intellectual property/registration matters relevant to the work.

4. Comparison Across Systems

To appreciate the distinct role of Zero-Centric Arithmetic, it is necessary to place it side by side with other perspectives. Traditional field theory, the simple logic of distribution, and the proposed framework each approach division by zero in different ways, with implications for consistency, philosophy, and pedagogy. The following comparison highlights how the Zero-Centric view avoids contradictions while offering a clearer and more unified interpretation.

4.1 Introductory Note

To highlight the distinct contribution of the Zero-Centric framework, we compare it with the traditional field-theoretic view and the simple logic of distribution. Table 1 summarizes the three perspectives across core aspects: definition, the status of zero, treatment of division by zero, contradictions, and philosophical stance.

Table 1. Comparative Perspectives on Division by Zero

Aspect	Traditional Field Theory	Simple Distribution Logic	Zero-Centric Arithmetic (Proposed)
Definition of Division	$a \div b = a \times b^{-1}$, $b \neq 0$	Distribute a among b persons	Division exists only if $b \neq 0$
Status of Zero	Exception, no inverse	No divisor present	Central element, excluded from division
Division by Zero	Undefined	Non-existent (No operation)	Non-existent (Operation never arises)
Contradictions	Appear if forced	None	None
Philosophical View	Deficient element	Absence of divisor	Center of numerical system

4.2 Discussion of Table 1

\This comparison shows that while traditional field theory leaves contradictions and simple distribution logic offers partial intuition, Zero Centric Arithmetic delivers a coherent framework grounded in the non-existence of the operation itself whenever the divisor vanishes.

4.3 Broader Cross-Disciplinary Perspectives

To clarify practical implications, Table 2 compares representative cases across mathematics, physics, engineering, programming, databases, and AI. In each domain, the No Operation principle eliminates contradictions by refusing to treat infinity as a numerical outcome.

Table 2. Comparative Perspectives on Division by Zero Across Disciplines

Domain	Example	Traditional Interpretation	Zero-Centric (No Operation)
Mathematics	$\left(\frac{1}{0}\right), \left(\frac{\Delta x}{\Delta y}\right)$ with $\Delta x = 0$	Undefined / Limits	No operation (no slope for vertical line)
Signals & Systems	$\left(H(f) = \frac{1}{j2\pi f}\right)$ f=0	Excluded / Approximation	No response (No Operation)
Digital Communications	$\left(SNR = \frac{Psignal}{Pnoise}\right)$, Pnoise = 0	Undefined	No operation (no SNR without noise)
Information Theory	$H(X) = -\Sigma p(x) \log p(x)$, $H(X) = p(x) = 0$	$Limit \rightarrow 0$	Absent term No Operation)
Wireless Channels	$\left(y = \frac{x}{h}\right), h = 0$	Undefined	No operation (no channel)
Programming	$\left(\frac{1}{0}\right)$ in Python or C/Java	Error (Zero Division Error)	Return No Operation
Databases	Division by zero in SQL	NULL / Query Fail	Return No Operation
Artificial Intelligence	Normalization: $\left(\frac{x}{\sigma}\right), \sigma = 0$	Add small ε	Skip operation (No Operation)
Physics	$\left(F = \frac{kq_1q_2}{r^2}\right), r = 0$	Undefined (Singularity)	No operation (no force at $r = 0$)
Cosmology	$\left(\rho = \frac{m}{v}\right), v = 0$	Singularity	No operation (no physical state)

5. Contradictions in Limits (Denominator → 0)

This subsection frames why division at a vanishing denominator should not be treated as yielding values. While (1/x) exhibits divergent one-sided limits near $x = 0$, these limits describe **behavior**, not results; treating ∞ as a number invites contradictions and indeterminate forms. The Zero Centric view resolves this by classifying every denominator $(x \rightarrow 0)$ instance as **No Operation**(a process that never arises) an interpretation the following examples and abacus demonstration make explicit.

1) Existence versus non-existence. At $x = 0$, the expression:

$$f(x) = \left(\frac{1}{x}\right) \quad (8)$$

is undefined. Yet, as $x \rightarrow 0^{+}$ the limit is written as:

$$\lim_{x \to 0^{+}} \left(\frac{1}{x}\right) = +\infty$$

and as $x \rightarrow 0^{-}$ the limit is written as:

$$\lim_{x \to 0^{-}} \left(\frac{1}{x}\right) = -\infty$$

Equation (8) therefore represents a contradiction: the operation both does not exist and simultaneously produces infinite outcomes.

2) Directional inconsistency. The one-sided limits of Equation (8) differ, implying there is no single value. Nevertheless, infinity is often informally used as though it were a valid numerical result.

3) Algebra with infinity. Expressions such as:

$$\infty - \infty$$

are considered indeterminate, yet they appear in manipulations that obscure logical structure.

4) Resolution under Zero-Centric Arithmetic. Instead of treating these divergences as values, the Zero-Centric view interprets the entire situation "denominator $\infty \rightarrow 0$" as No Operation. The differing behaviors are understood not as results, but as indicators that division never arise.

5.1 Illustrative Examples

1) From Equation (8), the classical view assigns $+\infty$ (right) and $-\infty$ (left), but the Zero-Centric interpretation asserts that the division itself never exists at $x = 0$.

$$f(x) = \left(\frac{1}{x^2}\right) \quad (9)$$

Classical analysis states $\lim_{x \to 0^{+}} \left(\frac{1}{x}\right) = +\infty$ and $\lim_{x \to 0^{-}} \left(\frac{1}{x}\right) = -\infty$. Yet, under the Zero-Centric view, Equation (9) never arises at $x = 0$ because the divisor vanishes.

$$f(x) = \left(\frac{\sin x}{x}\right) \quad (10)$$

Traditionally, $\lim_{x \to 0} f(x) = 1$, derived by series or geometric reasoning. However, Equation (10) demonstrates that at $x = 0$ the division does not exist. The analytic redefinition is valid as a new function value, not as the completion of a division.

5.2 Physical Demonstration Using the Libyan American Abacus (U.S. Patent No. 12,204,361 B2)

To provide a tangible validation of the Zero-Centric hypothesis, a physical proof can be observed through the Libyan American Abacus, an educational device patented under *U.S. Patent No. 12,204,361 B2*.

The abacus demonstrates division as an action of distributing quantities across activated bead groups:

Case 1: 24 ÷ 4 = 6:

As shown in Fig. 1, the dividend 24 is represented by activating beads in the tens and unit's columns (the two left columns), while the divisor 4 is represented by reference beads in the right column. Since the upper beads each represent value 2, beads are then activated sequentially in multiples of four $(8 + 8 + 8 = 24)$, confirming the division result. The visual process requires doubling the divisor representation first by conceptually moving the lower bead downward to visualize it as 8, then repeating the same for the next upper beads until the total reaches 24. Finally, these beads are read as a normal abacus configuration to confirm the quotient 6.

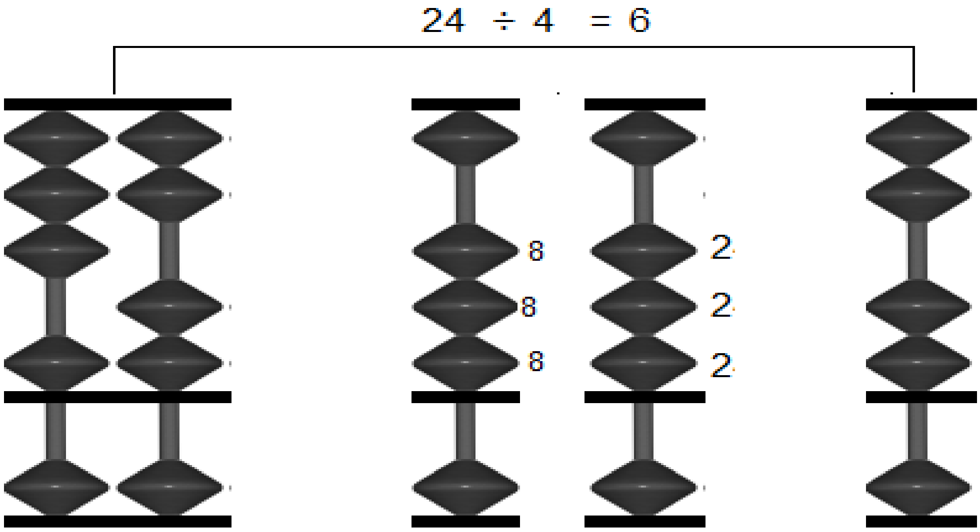

Fig. 1. Division Demonstration Using the Libyan-American Abacus

Case 2: 24 ÷ 0:
The number 0 is represented by *no activated beads*. Because no groups can be formed, no multiples arise, and no beads move. Consequently, no division takes place at all, there is no operation, not even a failed one.
This visual and physical representation corroborates the Zero-Centric interpretation: division by zero is not merely undefined but nonexistent, as the process itself cannot begin without a divisor.

6. Practical Examples

1) Gift distribution:

$$12 \div 3 = 4,\ 12 \div 1 = 12,\ 12 \div 0 = \text{No Operation} \qquad (11)$$

Equation (11) shows that while division is meaningful with recipients, the case of zero recipients corresponds to No Operation. This is consistent with the earlier contradiction introduced in Equation (2).

2) Geometry:

$$slope = \mathrm{m} = \left(\frac{\Delta \mathrm{y}}{\Delta \mathrm{x}}\right) \qquad (12)$$

Equation (12) works in most cases, but for vertical lines where $\Delta\mathrm{x} = 0$, the expression collapses into a division by zero. Just as with Equations (8) and (9), the Zero-Centric view declares this a No Operation. In this framework, the slope simply does not exist, and vertical lines have no slope in this sense.

3) Calculus usage:
The expression

$$\lim_{x \to 0} \left(\frac{1}{x}\right) \qquad (13)$$

is often described as unbounded growth, producing +∞ or −∞. Yet, consistent with Equations (8), (9), and (10), the Zero-Centric interpretation holds that at $x = 0$ the division never exists. Equation (13) therefore reflects the impossibility of the operation rather than a legitimate numerical output.

7. Philosophical and Educational Implications

The implications of the Zero Centric view are threefold:
1) Education: removes ambiguity for learners by replacing undefined with No Operation if there are no groups, there is no division.
2) Philosophy: zero is a center representing absence, not a flaw; it does not require an inverse where the operation is absent.
3) Mathematics and engineering encourage designs that eliminate singular branches by interpreting them as non-operations or domain exclusions.

8. Future Work

This paper introduces a Zero-Centric perspective on division by zero, positioning it as an operation fundamentally undefined rather than infinitesimally divergent. While foundational in nature, this reinterpretation invites a wide array of future investigations that span theoretical mathematics, pedagogical approaches, and computational modeling.

1. **Formal Axiomatic Integration**: Future research may focus on incorporating the Zero-Centric framework into formal axiomatic systems, especially those that extend or modify Zermelo-Fraenkel set theory or Peano arithmetic. This would enable rigorous comparison between conventional and Zero-Centric treatments of undefined operations.

2. **Algebraic Structures with Zero-Centric Constraints**: Developing algebraic systems (e.g., Zero-Centric fields or rings) where division by zero is explicitly ruled out or redefined with new axioms could yield consistent structures for abstract algebra and computational algebra systems.

3. **Computational Safeguards and Simulations**: Inspired by the Zero-Centric model, future work could implement computational systems (e.g., programming languages, symbolic solvers) that issue alerts or context-sensitive warnings when division by zero is approached, integrating it into error-detection and symbolic reasoning engines.

4. **Zero-Centric Pedagogy**: The reinterpretation offers potential value in mathematics education by shifting from the traditional "forbidden" model to a more explanatory, operationally grounded framework. Further research is needed to design and assess curriculum tools that incorporate Zero-Centric reasoning for middle and high school learners.

5. **Philosophical and Logical Extensions**: Given the unresolved metaphysical questions around the status of zero and nothingness, further philosophical work may explore how Zero-Centric Arithmetic interacts with formal logic, metaphysics of absence, and frameworks like intuitionism or constructivism.

6. **Extended Limit Analysis and Topological Models**: Future studies may explore whether Zero-Centric principles can be embedded into topological or analytical frameworks, such as via modified limit definitions, generalized functions, or non-standard analysis, to model how functions behave near undefined zones without invoking infinity.

9. Conclusion

Zero-Centric Arithmetic represents a fundamental shift in mathematical perspective. It replaces the traditional interpretations of undefined and infinity as a value with a principled stance that division simply does not arise when the divisor vanishes. This approach resolves long-standing contradictions, clarifies pedagogy, and opens practical pathways for application across disciplines.

Adopting this framework transforms undefined into non-existent and establishes simple logic as the solid foundation of arithmetic. Zero is no longer seen as a deficient element lacking an inverse, but rather as the central element of the number system an element that neither divides nor can be divided upon yet retains its pivotal role in the structure of numbers. In this way, Zero Centric Arithmetic becomes the basis for correcting misconceptions, simplifying mathematics, and rendering it more coherent and logical.

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