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Solving Nonlinear Volterra-Fredholm Integro-Differential Equations of the Second Kind by Combining the Sumudu Transform and the Adomian Decomposition Method

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Aabstract

In this paper, we modify the Sumudu decomposition method, the Sumudu Adomian Decomposition Method (SADM) which is a combination of the Sumudu transform method with the modified Adomian method and apply this method to solve mixed Fredholm-Volterra integro-differential equations, To study the performance of this technique, we introduced tow Numerical Examples of nonlinear Fredholm-Volterra integro-differential equations , The result shows that the proposed method is simple, fast and easy to implement.

1 INTRODUCTION

There are two possibilities for the limits of integration, if the limits of integration are known and constant, the integration is called the Fredholm integral equation, and if one of the limits of integration is a variable, it is called the Volterra integral equation, If the unknown function appears under the integral sign.

The equation is of the first kind, and if the unknown function appears under the integral sign and outside the integral sign, then the equation is of the second type and If the integral equation contains a derivative of the unknown function, it is called an integro-differential The linear and nonlinear Fredholm-Volterra integral equations solve many problems of mathematical physics, problems of population mechanics, etc (Abdou, 2003), (Abdou, 2004), (Hassan. Et al., 2010).in many scientific fields. A variety of analytical and numerical methods have been used to solve the Volterra-Fredholm integral equations, and obtaine the solution of these integral equations analytically and numerically using one of the following methods(Ayyubi ,2021),(Abdul, 1997), Homotopy perturbation method , ationalized Haar functions Taylor polynomial method, Adomian decomposition method and Minggen et, The basic ideas and earlier work presented by Adomian and Wazwaz are applied to a nonlinear arrangement of Fredholm-Volterra integro-differential equations (Tavassoli et al., 2007) can also use the Sumudu transform to solve many similar problems and apply it to many regular and partial differential equations This transformation can also be applied to the neutron transport equation (Esmail i et al., 2012) . In this paper some basic properties of the Sumudu transform and the transform efficiency in solving Fredholm-Volterra integral equations are determined.

2-DEFINITION OF SUMUDU TRANSFORM (Ahmad et al., 2015)

The Sumudu transform of the real function $F(t)$ for all $t \geq 0$ is defined as

$$S_u[f(t)] = F[u] = \int_0^\infty f(ut)e^{-t}dt, \quad u\epsilon(-\tau_1, \tau_2) \quad (2.1)$$

Table 1. Special Sumudu transforms (Hassan et al., 2010), (Hradyesh, 2015)

$f(t)$	$F(u) = S_u[f(t)]$
1	1
t	u
$t^n, n \in N$	$n!u^n$
e^{at}	$\frac{1}{1-au}$
$\sin at$	$\frac{au}{1+a^2u^2}$
$\cos at$	$\frac{1}{1+a^2u^2}$
$\sinh at$	$\frac{au}{1-a^2u^2}$
$\cosh at$	$\frac{1}{1-a^2u^2}$

Table 2. Sumudu transform fundamental properties (Hassan et al., 2010),

Name of property	Mathematical Form
Linearity	$S_u[a f(t)+ bg(t)] = a S_u[f(t)] + bS_u[g(t)]$
Sumudu transform convolution theorem	$S_u\left[\int_0^t f(t)\, g(t-\tau)d\tau\right] = uF[u]G[u]$
First- scale preserving theorem	$S_u[f(at)] = F[au]$
Second - scale preserving theorem	$S_u\left[t\frac{df(t)}{dt}\right] = u\frac{dF[u]}{du}$
Duality with laplace transform	$F[u] = \frac{F_L[1/u]}{u}$ and $G[u] = \frac{G[1/s]}{s}$
Shifting	$S_u[e^{at} f(t)] = \frac{1}{1-au}F[\frac{u}{1-au}]$
Sumudu transform of an integral of a function	$\int_0^t f(\tau)d\tau = uF(u)$
Sumudu transform of function derivative	$S_u[f\acute{}(t)] = F\acute{}[u] = \frac{F[u]}{u} - \frac{f(0)}{u}$, $S_u\left[f\acute{\acute{}}(t)\right] = F\acute{\acute{}}[u] = \frac{F[u]}{u^2} - \frac{f\acute{}(0)}{u^2} - \frac{f(0)}{u}$ $S_u[f^{(n)}(t)] = F^{(n)}[u] = \frac{F[u]}{u^n} - \sum_{j=0}^{n-1}\frac{f^{(j)}(0)}{u^{n-j}}$

3- The Adomian Decomposition Method (Ebrahim et al., 2009) , (Rezvan et al., 2018)

The Adomian decomposition method is applied to many nonlinear equations. Consider the general nonlinear Fredholm-Volterra integro differential equations

$$y^{(n)}(t) = h(t) + \lambda_1 \int_a^t k_1(t,x)\, G_1\big(y(x)\big)dx + \lambda_2 \int_c^d k_2(t,x)\, G_2\big(y(x)\big)dx. \quad (3.1)$$

Where $a, b, d,$ are constant values, $h(t)$ is known function of t, $k_1(t,x)$, $k_2(t,x)$ are known functions of t and x called the kernels, $G_1\big(y(x)\big)$, $G_2\big(y(x)\big)$ are nonlinear functions of $y(x)$, λ_1, λ_2 are scalar parameters or constants and $y^{(n)}(t) = d^{(n)}\, y(t)/dt^n$.

The first step of Adomian method is to represent the solution $y(t)$ of the n^{th} order non-linear Fredholm-Volterra integro-differential equation (3.1) by the infinity series

$$y = \sum_{k=0}^{\infty} y_k \qquad (3.2)$$

The nonlinear term is decomposed into

$$G_1(y) = \sum_{k=0}^{\infty} A_k \,, G_2(y) = \sum_{k=0}^{\infty} B_k \quad (3.3)$$

Where A_k , B_k ; $k = 0,1,2,...$ are special polynomials known as Adomian polynomials. The Adomian polynomials were introduced in term of the Kronecker delta $\delta_{n,m}$, with $k \geq v$ as $A_0 = G_1(y_0)$ for $k = 1,2,...$ by

$$A_k = \sum_{v=1}^{k} [\frac{1}{v!} \sum_{i_1,i_2,...i_v=1}^{k+1-v} \delta_{k,i_1+i_2+...+i_v} y_{i_1} y_{i_2} ... y_{i_v}] \frac{d^v G_1(y_0)}{dy_0^{\,v}} , \qquad (3.4)$$

By using a similar manner in(3.4), the polynomials $B_0 = G_2(y_0)$ can be written in the form

$$B_k = \sum_{v=1}^{k} [\frac{1}{v!} \sum_{i_1,i_2,...i_v=1}^{k+1-v} \delta_{k,i_1+i_2+...+i_v} y_{i_1} y_{i_2} ... y_{i_v}] \frac{d^v G_2(y_0)}{dy_0^{\,v}} , \qquad (3.5)$$

Hence, for $k = 0,1,2,3$, we have $A_0 = G_1(y_0)$, $B_0 = G_2(y_0)$ and the recurrent formula (3.3) gives

$$A_1 = y_1 G_1'(y_0)$$

$$A_2 = y_2 G_1'(y_0) + \frac{1}{2!} y_1^2 G_1''(y_0) \qquad (3.6)$$

$$A_3 = y_3 G_1'(y_0) + y_1 y_2 G_1''(y_0) + \frac{1}{3!} y_1^3 G_1'''(y_0)$$

$$B_1 = y_1 G_2'(y_0)$$

$$B_2 = y_2 G_2'(y_0) + \frac{1}{2!} y_1^2 G_2''(y_0) \qquad (3.7)$$

$$B_3 = y_3 G_2'(y_0) + y_1 y_2 G_2''(y_0) + \frac{1}{3!} y_1^3 G_2'''(y_0)$$

Before we apply the standard Adomian method, let us write an expression for $y(t)$ that will be derived from (3.1). We do that by integrating both sides of (3.1) n- times from 0 to x . Consequently, we can obtain

$$y(t) = \sum_{i=0}^{n-1} \frac{a_i}{i!} t^i + L^{-n}\big(h(t)\big) + \lambda_1 \int_a^t k_1(t,x) G_1\big(y(x)\big)dx + \lambda_2 \int_c^d k_2(t,x) G_2\big(y(x)\big)dx, \quad (3.8)$$

,

Where L^{-n} is an n- fold integration operator. Now substituting (3.2), and (3.3) into (3.8) yields

$$\sum_{K=0}^{\infty} Y_k(t) = \sum_{i=0}^{n-1} \frac{a_i}{i!} t^i + L^{-n}\big(h(t)\big) + \lambda_1 \int_a^t k_1(t,x) \sum_{k=0}^{\infty} A_k dx + \lambda_2 \int_c^d k_2(t,x) \sum_{k=0}^{\infty} B_k dx \quad (3.9)$$

This leads to the recursive relations

$$\boldsymbol{y_o(t)} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{n-1}} \frac{\boldsymbol{a_i}}{\boldsymbol{i}!}$$

$$\mathrm{y}_0(\mathrm{t}) = \sum_{\boldsymbol{i=0}}^{\boldsymbol{n-1}} \frac{\boldsymbol{a_i}}{\boldsymbol{i}!} \boldsymbol{t^i} + L^{-n}\big(h(t)\big)$$

$$Y_{k+1}(t) = \lambda_1 L^{-n}\left(\int_a^t k(t,x) A_k dx\right) + \lambda_1 L^{-n}\left(\int_c^d k(t,x) B_k dx\right) k = 0,1,2,.... \quad (3.10)$$

Hence, all the components $y_k(x)$, $k = 0,1,2,3$ can be found from the recursive relation (3.9), once the polynomials A_k , B_k are calculated. Finally, we define the K -terms approximant by $\tilde{\mathrm{y}}_\mathrm{k} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{k}} \boldsymbol{y_i}$ and assume that$\mathrm{y(t)} = \lim_{\mathrm{k}\to\infty} \{\tilde{\mathrm{y}}_\mathrm{k}\}$ (3.1)

4-SUMUDU-ADOMIAN DECOMPOSITION METHEOD (Maleknejad & Hadizadeh, 1999)

The Sumudu-Adomian method (SAM) is a combination of theSumudu transform method and the standard Adomian method, where the standard Adomian method was developed for the analytical treatment of nonlinear terms using Adomian polynomials.To review this method, we assume that the kernel of equation (3.1) is a difference kernel and apply the Sumudu transformation to both sides of equation (3.1).
This leads to

$$S_u\left[\left(y^{(n)}(t)\right)\right] = S_u[h(t)]+\lambda_1 S_u\left[\int_a^t k_1(t-x)\, G_1\big(y(x)\big)dx\right]+\lambda_2 S_u\left[\int_c^d k_2(t-x)\, G_2\big(y(x)\big)dx\right] \quad (4.1)$$

Then using the derivative and the convolution theorem, (4.1) is reduced to the form (3.8), which allows us to write

$$S_u[y(t)] = u^n \sum_{j=0}^{n-1} \frac{y^{(j)}(0)}{u^{n-j}} + u^n S_u[h(t)] + \lambda_1 u^{n+1}\big(S_u[k_1(x)]S_u[G_1(y(x))]\big) + \lambda_2 u^{n+1}\Big(S_u[k_2(t)]S_u[G_2(y(x))]\Big), \quad (4.2)$$

and by setting $S_u[k_1(x)] = K_1(u)$, $S_u[k_2(x)] = K_2(u)$ we canwrite (4.2) in the form

$$S_u\Big[\sum_{k=0}^{\infty} y_k\Big] = u^n \sum_{j=0}^{n-1} \frac{y^{(j)}(0)}{u^{n-j}} + u^n S_u[h(x)] + \lambda_1 u^{n+1}\Big(K_1(u) S_u\Big[\sum_{k=0}^{\infty} A_k\Big]\Big) + \lambda_2 u^{n+1}\Big(K_2(u) S_u\Big[\sum_{k=0}^{\infty} B_k\Big]\Big) \quad (4.3)$$

$$S_u[y_0] = u^n \sum_{j=0}^{n-1} \frac{y^{(j)}(0)}{u^{n-j}} + u^n S_u[h(X)] \quad (4.4)$$

This leads to the recursive relation

$$S_u[y_{k+1}] = +\lambda_1 u^{n+1}\Big(K_1(u) S_u[A_k(y_0, y_1, ..., y_k)]\Big) + \lambda_2 u^{n+1}\big(K_2(u) S_u[B_k(y_0, y_1, ..., y_k)]\big),$$
$$k = 1, 2, ... \quad (4.5)$$

At theend, applying the inverse Sumudu transform to both sides of (4.4) gives y_0, and making use of the recursive relation (4.5) enables us to find the components y_k for $k = 1, 2, ...$.

5-THE COMBINED SUMUDU MODIFIED ADOMIANECOMPOSITION METHEOD

The general form of non-linear Fredholm-Volterra integro- differential equations can be expressed as following (Ahmed, 2016),(Fariborzi & Sadigh, 2009)

$$y^{(n)}(t) = h(t) + \lambda_1 \int_a^t k_1(t,x)\, G_1\big(y(x)\big)dx + \lambda_2 \int_c^d k_2(t,x)\, G_2\big(y(x)\big)dx. \quad (5.1)$$

Where $h(x)$ is known function of x , $k(x,t)$ is known function of x and t called the kernel, $G_1(y(t))$ is known scalar λ and $y^{(n)}(x) = d^{(n)}\, y(x)/dt^n$, $y(t)$ of nonlinearare a $G_2(y(t))$, parameter or constant.

Applying the Sumudu transform to both sides of (5.1) gives

$$S_u\left[\left(y^{(n)}(t)\right)\right] = S_u[h(t)]+\lambda_1 S_u\left[\int_a^t k_1(t-x)\,G_1\big(y(x)\big)dx\right] + \lambda_2 S_u\left[\int_c^d k_2(t-x)\,G_2\big(y(x)\big)dx\right] \tag{5.2}$$

On the other hand, using the differentiation and convolution property of Sumudu transform, we have

$$S_u\left[y(t)\right] = u^n \sum_{j=0}^{n-1}\frac{y^{(j)}(0)}{u^{n-j}} + u^n S\left[h(t)\right] + \lambda_1 u^{n+1} S_u\left[k_1(x)\right] S_u\left[G_1 y(x)\right] + \lambda_2 u^{n+1} S_u\left[k_2(x)\right] S_u\left[G_2 y(x)\right] \tag{5.3}$$

Represent the solution as in infinite series given by

$G_1 y(x) = \sum_{k=0}^{\infty} A_K$ and $G_2 y(x) = \sum_{k=0}^{\infty} B_K$

On comparing both side of (5.3) and by using standard (ADM)

$$S_u[y(t)] = u^n \sum_{j=0}^{n-1}\frac{y^{(j)}(0)}{u^{n-j}} u^n S_u[h(t)]\lambda_1 u^{n+1}\left(K_1(x)S_u\left[\sum_{k=0}^{\infty} A_k\right]\right)+\lambda_2 u^{n+1}\left(K_2(x)S_u\left[\sum_{k=0}^{\infty} B_k\right]\right), \tag{5.4}$$

Thus the following recursive relations for the modified Adomian decomposition method are formulated as

$$\boldsymbol{S_u}[y_0] = u^n \sum_{j=0}^{n-1}\frac{y^{(j)}(0)}{u^{n-j}} \tag{5.5}$$

This leads to the modified recursive relation

$$\boldsymbol{S_u}[y_{k+1}] = u^n \boldsymbol{S_u}[h_k(t)] + \lambda_1 u^{n+1}\Big(K_1(u)\boldsymbol{S_u}[A_k(y_0, y_1, ..., y_k)] + \lambda_2 u^{n+1}\Big(K_2(u)\boldsymbol{S_u}[B_k(y_0, y_1, ..., y_k)]\Big)\Big)$$

$$, k = 0,1,2,... \tag{5.6}$$

At the end, applying the inverse Sumudu transform to both sides of (5.5) gives y_0, and making use of the relation (5.6) enables us to find the components y_k for $k \geq 0$.

6- Numerical Examples

In this part two examples are provided. These examples are considered to illustrate ability and reliability of the new method. .

Example (1): (Salıh**,** 2002) Consider the 1^{st} order nonlinear volterra-Fredholm integro-differential equation with

$$\acute{y}(t)=\frac{9}{4}-\frac{5}{2}t-\frac{1}{2}t^2-3e^{-t}-\frac{1}{4}e^{-2t}+\int_0^t (t-x)y^2(x)dx, k_2=0 \qquad y(0)=3 \qquad (6.1)$$

Subject to the initial conditions $y(0)=2$ .Hence comparing (6.1) and the nonlinear integro-differential equation,

To check the performance of the modified Sumudu-Adomian method, we first write the Taylor series expansion of $h(t)$. These reads

$$h(t)=\sum_{i=0}^{\infty} h_i(t)=-1+t-\frac{5}{2!}t^2+\frac{5}{3!}t^3+\dots$$

Next by using (5.5), we can set $S_u[y_0]=0$. This leads to $y_0=2$.

For $k=0$, the recursive relation (5.4) reads

$$S_u[y_1]=uS_u[-1]+u^2\left[uS_u[2]^2\right]=-u+4u^3 \qquad (6.2)$$

And the inverse Sumudu transform table, ref [6], gives $y_1=-t+\frac{4}{3!}t^3$.

Similarly, for $k=1$, (5.6) reads

$$S_u[y_2]=uS_u[t]+u^2\left(u\,S_u[-4t+\frac{16}{3!}t^3]\right)=u^2-4u^4+16u^5,$$

And the inverse Sumudu transform table gives $y_2=-\frac{t^2}{2!}-\frac{4t^4}{4!}+\frac{16t^5}{5!}$

Also for $k=2$ and $k=3$, the recursive relation (5.6) reads

$$S_u[y_3]=uS_u\left[-\frac{5}{2!}t^2\right]+u^2\left[uS_u\left[\frac{6t^2}{2!}-\frac{48t^4}{4!}+\frac{64t^5}{5!}+\frac{320t^6}{6!}\right]\right]$$

,

$$=-5u^3+6u^5-48u^7+64u^8+320u^9 \qquad (6.3)$$

$$S_u[y_4]=u^2S_u[-\frac{2}{5!}t^6]-u^3\left(u\,S_u[\frac{8t^6}{6!}+\frac{4t^8}{8!}+\frac{2t^{10}}{10!}]\right)$$

$$=-12u^8-8u^{10}-4u^{12}-2u^{14} \qquad (6.4)$$

Respectively Hence applying the inverse Sumudu transform on (6.3) and (6.4) respectively we can find

$$y_3 = \frac{-5}{3!}t^3 + \frac{6}{5!}t^5 - \frac{48}{7!}t^7 + \frac{64}{8!}t^8 + \frac{320}{9!}t^9 \quad (6.5)$$

$$y_4 = -\frac{12t^8}{8!} - \frac{8t^{10}}{10!} - \frac{4t^{12}}{12!} - \frac{2t^{14}}{14!} \quad (6.6)$$

Finally, if we continue the same procedure for $k = 4,5,\dots,$ we can compute the k-term approximant $\tilde{y}_k(t)$ of the solution $y(t)$, which leads to the Taylor series expansion of the exact solution.

$$y(t) = 2 - t + \frac{1}{2!}t^2 - \frac{1}{3!}t^3 + \frac{1}{4!}t^4 \quad (6.7)$$

that converges to the exact solution $y(t) = 1 + e^{-x}$

Example (2) (Rezvan et al., 2010)

Consider the 1^{st} order nonlinear Volterra-Fredholm integro differential equation with

$$\acute{y}(t) = 2e^{2t} - \frac{1}{24}e^t + \frac{1}{24} + \int_0^1 e^{t-4x}y^2(x)dx, k_1 = 0, \quad k_2 = e^{t-4x} \qquad y(0) = 2 \qquad (6.8)$$

Subject to the initial conditions $y(0) = 1$.The Taylor series expansion of $h(t)$ as

$$h(t) = \sum_{i=0}^{\infty} h_i(t) = \left(2 - \frac{1}{24}\right) + \left(4 - \frac{1}{24}\right)t + \left(8 - \frac{1}{24}\right)\frac{t^2}{2!} + \left(16 - \frac{1}{24}\right)\frac{t^3}{3!} \qquad (6.9)$$

To apply the modified Sumudu-Adomian method, we write $S_u[y_0] = 1$. This gives $y_0 = 1$.
To find the next component, we use the recursive relation (5.6), with $A_0(y_0) = y_0^2$. This gives

$$S_u[y_1] = uS_u\left[\left(2 - \frac{1}{24}\right)\right] + \frac{u^2}{24}\left[\frac{1}{1-u}S_u[(1)^2]\right] = \left[2 - \frac{1}{24}\right]u + \frac{1}{24}\frac{u^2}{1-u}$$

, (6.10)

Then the inverse Sumudu transform table gives

$$y_1 = \left[\left(2 - \frac{1}{24}\right)\right]t + \frac{1}{24}\frac{t^2}{2!}e^t, \qquad (6.11)$$

For $k = 1$, the recursive relation (5.6) reads

$$S_u[y_2] = uS_u[h_1(t)] + u^2\big(K(u)\ S_u[A_1(y_0)]\big),$$

$$,\ A_1(y_0) = 2y_0 y_1.\ \text{gives}$$

$$S_u[y_2] = uS_u\left[\left(4-\tfrac{1}{24}\right)t\right] + \tfrac{u^2}{24}\left[\tfrac{1}{1-u}\right]S_u\left[2\left(2-\tfrac{1}{24}\right)t + \tfrac{2}{24}\tfrac{t^2}{2!}e^t\right] \qquad (6.12)$$

Similarly, using the inverse Sumudu transform table gives

$$y_2 = \left[\left(4-\frac{1}{24}\right)\right]\frac{t^2}{2!} + \frac{2}{24}\left(2-\frac{1}{24}\right)\frac{t^3}{3!}e^t + \frac{2}{(24)^2}\frac{t^3}{3!}(te^t)$$

For $k = 2$, then recursive relation (5.6) reads

$$S_u[y_3] = uS_u[h_2(t)] + u^2\big(K(u)\ S_u[A_2(y_0)]\big),\ A_2(y_0) = 2y_0 y_2 + y_1^2$$

This leads to

$$S_u[y_3] = uS_u\left[\left(8-\tfrac{1}{24}\right)\tfrac{t^2}{2!}\right]\tfrac{u^2}{24}\left[\tfrac{1}{1-u}S_u\left[2\left(4-\tfrac{1}{14}\right)\tfrac{t^2}{2!} + \tfrac{4}{24}\left(2-\tfrac{1}{24}\right)\tfrac{t^3}{3!}e^t + \right.\right.$$
$$\left.\left.\tfrac{4}{(24)^2}\tfrac{t^3}{3!}(te^t) + 2\left(2-\tfrac{1}{24}\right)^2\tfrac{t^2}{2!} + \tfrac{1}{4}\left(2-\tfrac{1}{24}\right)\tfrac{t^3}{3!}e^t + \tfrac{1}{96}\tfrac{t^4}{4!}(e^t)^2\right]\right] (6.13)$$

The inverse Sumudu transform table gives

$$y_3 = \left(8-\frac{1}{24}\right)\frac{t^3}{3!} + \frac{2}{24}\left(4-\frac{1}{24}\right)\frac{u^4}{1-u} + \frac{4}{24}\left(2-\frac{1}{24}\right)\frac{u^5}{(1-u)^2} + \frac{4}{(24)^2}\frac{u^6}{(1-u)^3}$$
$$+\frac{2}{24}\left(2-\frac{1}{24}\right)^2\frac{u^4}{(1-u)} + \frac{1}{24}\frac{1}{4}\left(2-\frac{1}{24}\right)\frac{u^5}{(1-u)^2} + \frac{1}{24}\frac{1}{96}\frac{u^6}{(1-u)^3}$$
(6.14)

Finally, with the help of the sports packages,we can continue the same procedure for $k = 4,5,6,7,\ldots$. From the above results, we can conclude that the k -terms approximant $\varphi_k(t)$ of the solution $y(t)$, will lead to the Taylor series expansion of the exact solution, which is given by

$$y(t) = 1+2t+\frac{1}{2!}(2t)^2 + \frac{1}{3!}(2t)^3 + \ldots .6.15)$$
$$y(t) = e^{2x}(6.16)$$

4 CONCLUSIONS:

In this paper, we combine the sumudu transform with the Adomian method (ADM) and present an approximate method for solving nonlinear Fredholm-Volterra integro-differential equations using the sumudu-Adomian decomposition method. To investigate the performance of the modified approach, we study the solutions of special types of integro-differential equations when the homogeneous limit of the integro-differential equations is complex.In addition, this method makes it easier to generate Adomian limits for the nonlinear limit, which reduces the amount of computations in each iteration.in addition, this method makes it easier to generate Adomian limits for the nonlinear limit, which reduces the amount of computations in each iteration.Finally, we can consider this method as a new contribution to the existing analytical, semi-analytical and numerical techniques.

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